Capital University of Economics and Business

ISEM

Financial Econometrics

Assignment 2

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Term:
Fall Semester, 2019

October 30, 2019

1 True/false questions

State whether each of the following statements is true or false.

Q1. An AR(1) process is always weakly stationary.

Q2. We can always estimate the coefficients in an AR process using the Yule-Walker equations.

Q3. The autocorrelation function of an AR(p) process cuts off at lag p.

Q4. The AR order selected by BIC tends to be smaller than that selected by AIC.

Q5. The variance of h-step-ahead forecast error increases with h in an AR process.

Q6. A model is adequate if the fitted residuals do not have serial correlation.

Q7. Any MA process can be written as an AR(∞) process.

Q8. An ARMA model is weakly stationary only if the MA part is invertible.

2 Multiple choice questions

Q1. Consider an AR(1) process $X_t = a_0 + a_1 X_{t-1} + \varepsilon_t$, where $\varepsilon_t \overset{\text{iid}}{\sim} \left(0, \sigma^2\right)$ and $|a_1| < 1$. Which of the following is true?

(A) X_t is weakly stationary.

(B) $\gamma_X(h) \to 0$ as $h \to \infty$.

(C) $\mathbb{E}\left[X_t\right] = 0$ if and only if $a_0 = 0$.

(D) All of the above.

Q2. Suppose you are trying to select the order of an AR(p) process. What is the suggested order according to the following table?

j	0	1	2	3	4
AIC(j)	0.066	-0.012	-0.039	-0.037	-0.034

(A) 0

(B) 1

(C) 2

(D) 3

Q3. Consider an AR(1) process $X_t = a_0 + a_1 X_{t-1} + \varepsilon_t$, where $\varepsilon_t \overset{\text{iid}}{\sim} (0, 1)$. Suppose also that the variance of X_t is $\gamma_X(0) = 2$. Find a_1.

(A) $\sqrt{0.5}$

(B) $-\sqrt{0.5}$

(C) All of the above.

(D) None of the above.

Q4. Consider an ARMA(1,1) process $X_t = a_1 X_{t-1} + \varepsilon_t - b_1 \varepsilon_{t-1}$, $\varepsilon \overset{\text{iid}}{\sim} \left(0, \sigma^2\right)$. Which of the following statements are equivalent?

(1) X_t is weakly stationary.

(2) $|a_1| < 1$.

(3) $|b_1| < 1$.

(4) $a_1 = b_1$.

(A) (1) and (2) only.

(B) (1) and (3) only.

(C) (1) and (4) only.

(D) (2) and (3) only.

3 Short questions

Q1. Forecasting an AR(1) process

Consider the AR(1) process

$$X_t = a_0 + a_1 X_{t-1} + \varepsilon_t, \qquad \varepsilon_t \overset{\text{iid}}{\sim} \mathcal{U}\left(-1, 1\right).$$

(a) Find the variance of a random variable with uniform distribution $\varepsilon \sim \mathcal{U}(-1, 1)$.

(b) Find the h-step-ahead forecast of X_t for $h = 1, 2$.

(c) What is the variance of the h-step-ahead forecast error?

Hint: The density function of a uniformly distributed function $\varepsilon \sim \mathcal{U}(a, b)$ is given by

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{otherwise.} \end{cases}$$

Q2. MA representation of an ARMA process

Consider the ARMA(1,1) process

$$X_t = a_1 X_{t-1} + \varepsilon_t - b_1 \varepsilon_{t-1}, \qquad \varepsilon_t \overset{\text{iid}}{\sim} \left(0, \sigma^2\right).$$

Express X_t as an MA(∞) process.