



CAPITAL UNIVERSITY OF ECONOMICS AND BUSINESS

ISEM

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## Financial Econometrics

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*Moments of the Uniform Distribution*

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Let  $X$  be a random variable with the density function

$$f_X(x) = \begin{cases} 1, & \text{if } x \in [0, 1] \\ 0, & \text{otherwise.} \end{cases}$$

Find the mean, variance, skewness and kurtosis of  $X$ .

**Mean** The mean of  $X$  is given by its expectation.

$$\mu_X = \mathbb{E}[X] = \int_0^1 x dx = \left[ \frac{x^2}{2} \right]_0^1 = \frac{1}{2}$$

**Variance** The variance of  $X$  is given by its second centered moment.

$$\begin{aligned} \sigma_X^2 &= \mathbb{E}[(X - \mu_X)^2] = \int_0^1 \left(x - \frac{1}{2}\right)^2 dx \\ &= \left[ \frac{1}{3} \left(x - \frac{1}{2}\right)^3 \right]_0^1 = \frac{1}{3} \left[ \left(\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^3 \right] \\ &= \frac{1}{12} \end{aligned}$$

**Skewness** The skewness of  $X$  is given by the third moment of the standardized variable.

$$\begin{aligned} S(X) &= \mathbb{E} \left[ \left( \frac{X - \mu_X}{\sigma_X} \right)^3 \right] = 12\sqrt{12} \int_0^1 \left(x - \frac{1}{2}\right)^3 dx \\ &= 24\sqrt{3} \left[ \frac{1}{4} \left(x - \frac{1}{2}\right)^4 \right]_0^1 = 6\sqrt{3} \left[ \left(\frac{1}{2}\right)^4 - \left(-\frac{1}{2}\right)^4 \right] \\ &= 0 \end{aligned}$$

**Kurtosis** The kurtosis of  $X$  is given by the fourth moment of the standardized variable.

$$\begin{aligned} K(X) &= \mathbb{E} \left[ \left( \frac{X - \mu_X}{\sigma_X} \right)^4 \right] = 144 \int_0^1 \left(x - \frac{1}{2}\right)^4 dx \\ &= 144 \left[ \frac{1}{5} \left(x - \frac{1}{2}\right)^5 \right]_0^1 = \frac{144}{5} \left[ \left(\frac{1}{2}\right)^5 - \left(-\frac{1}{2}\right)^5 \right] \\ &= \frac{9}{5} \end{aligned}$$