Applied Stochastic Process

4c Asset Returns

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Continuous Compounding

Effect of compounding

Suppose you are going to deposit \$10,000 in a bank, which offers you a 10% per annum interest rate and the following compounding scheme:

1. Compounding every year, where the one-year interest rate is 10%;
2. Compounding every 6 months, where the 6-month interest rate is $10\%/2 = 5\%$.

Which one should you choose?

Effect of compounding

Type	No. of payments	Interest rate per period	Total value
Annual	1	10%	$11000.00
Semiannual	2	5%	$11025.00
Quarterly	4	2.5%	$11038.13
Monthly	12	0.833%	$11047.13
Weekly	52	0.192%	$11050.65
Daily	365	0.027%	$11051.56

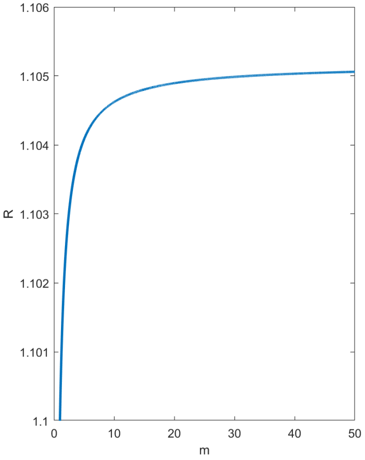
Table: Values of a loan with 10% per annum interest rate

Effect of compounding

In general, if the bank gives interest m times a year, you get

$$\$10,000 \times \left(1 + \frac{10\%}{m}\right)^m .$$

What if $m \to \infty$?



Continuous compounding

Suppose the continuously compounded interest rate is r, the simple gross return, or the *effective annual interest rate*, is

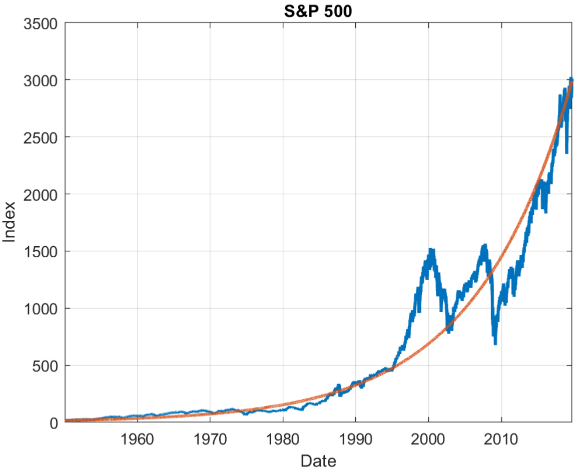
$$1 + R = \lim_{m\to\infty} \left(1 + \frac{r}{m}\right)^m .$$

Taking logarithm, and by L'Hopital's Rule

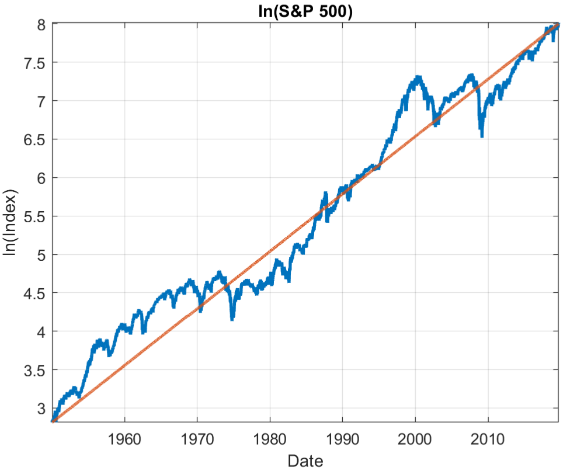
$$\lim_{m\to\infty} m \ln\left(1 + \frac{r}{m}\right) = r.$$

Therefore, $1 + R = \mathrm{e}^r$, or $r = \ln(1 + R)$, where r is also called the log return.

S&P 500 historical values



Logarithm of S&P 500



Stock Prices as Geometric Brownian Motion

Price process

Let r_t be the log-return of an asset, then the asset price at time t can be represented as

$$P_t = P_0 \mathrm{e}^{r_t}$$

Suppose further that r_t follows a Brownian motion with drift parameter μ and variance parameter σ^2. Then P_t follwos a geometric Brownian motion

$$P_t = P_0 \mathrm{e}^{\mu t + \sigma B_t}$$

where B_t is the standard Brownian motion.

Price process

The advantages of modeling the price process as a geometric Brownian motion are:

- Geometric Brownian motion takes only positive values.
- The rate of return P_t/P_{t-1} of asset price may be reasonably modeled as independently distributed.

Some criticisms are:

- Geometric Brownian motion does not take into account for extreme events.
- The observed returns are not normally distributed, e.g., there are excess kurtosis.