Applied Stochastic Process

Assignment 2 - Solutions

CHEUNG Ying Lun

12/4/2020

An epidemic caused by a new virus is spreading in City A. You are asked to evaluate the effect of several strategies in controlling the epidemic. Suppose the basic reproduction number (R_0) is estimated to be 2.3 (new cases from each old case per week). Three strategies are proposed to deal with the situation. The effective reproduction numbers (R_e) in the presence of control efforts are predicted to be 0.8, 1.2 and 1.8 respectively. Model the epidemic as a branching process and answer the following questions by means of simulation.

To simulate the branching process, we first set seed and assign values to the parameters.

```
####################################################
#Initialize
rm(list = ls())
graphics.off()
set.seed(1542)

####################################################
#Set parameters
NTrial <- 500
R0 <- 2.3
Re <- c(0.8, 1.2, 1.8)
InitialT <- 8
FinalT <- 13
```

The function for simulating a branching process is defined below:

```
####################################################
#Declare a function of branching process
branching <- function(R0, Re, T1, T2) {

  #Initialize
  NPath0 <- vector(mode = "integer", length = T1+1)
  NPath0[1] <- 1

  NPath1 <- matrix(nrow = T1+T2+1, ncol = length(Re))

  #Calculate the parameter of the Geometric distribution
  p0 <- 1 / (R0+1)
  pe <- 1 / (Re+1)
  t <- 1

  #Start simulation
  ########
  #Before controlling policies
```

```r
  while (t <= T1 & NPath0[t]!=0) {
    #NPath0[t+1] <- sum(rgeom(NPath0[t], p0))
    NPath0[t+1] <- rnbinom(1, NPath0[t], p0)
    t <- t+1
  }
  #Terminate if NPath0[t] == 0
  if (NPath0[t]==0) {
    NPath0[t:(T1+1)] <- 0
    NPath1[1:(T1+1), ] <- NPath0
    NPath1[(T1+2):(T1+T2+1), ] <- 0
    return(NPath1)
  }

  ########
  #After controlling policies
  for (iR in 1:length(Re)) {
    NPath1[1:(T1+1), iR] <- NPath0
    t <- 1
    while (t <= T2 & NPath1[t+T1, iR]!=0) {
      #NPath1[t+T1+1, iR] <- sum(rgeom(NPath1[t+T1, iR], pe[iR]))
      NPath1[t+T1+1, iR] <- rnbinom(1, NPath1[t+T1, iR], pe[iR])
      t <- t+1
    }
    if (NPath1[t+T1, iR]==0) {
      NPath1[(t+T1):(T1+T2+1), iR] <- 0
    }
  }

  return(NPath1)
}
```

Assuming that the number of new cases from each old case Z follows a geometric distribution with probability of success p. Note that here we assume Z to be the number of failures before the first success (the definition in R) instead of the number of trials until the first success (the definition in lecture notes), i.e., $p(Z) = p(1-p)^Z$. Note that R_0 and R_e are the expected values of a geometrically distributed random variable, we can deduce from the probability of success by the relation $p_0 = (R_0 + 1)^{-1}$ and respectively for p_e.

With X_{n-1} cases in total at time $n-1$, the number of new cases at time n is

$$X_n = \sum_{i=1}^{X_{n-1}} Z_{n,i}.$$

Therefore, we can simply calculate X_n as the sum of X_{n-1} geometrically distributed random variable, which can be done by the code `NPath0[t+1] <- sum(rgeom(NPath0[t], p0))`. However, if the process explodes, `rgeom(NPath0[t], p0)` can run very slowly. Therefore, we can use the fact that independent sum of geometrically distributed random variables is negative binomially distributed, and write `NPath0[t+1] <- rnbinom(1, NPath0[t], p0)`.

Finally, the number of new cases before and after the control policies can be obtained by running the above defined function.

```r
Ncases <- replicate(NTrial, branching(R0, Re, InitialT, FinalT))
```

1. Suppose the epidemic has already been left to grow without intervention for 8 weeks since the first case. What is the mean and variance number of new cases this week?

```r
print(paste("The mean number of new cases today is",
            mean(Ncases[InitialT, 1, ])))
```

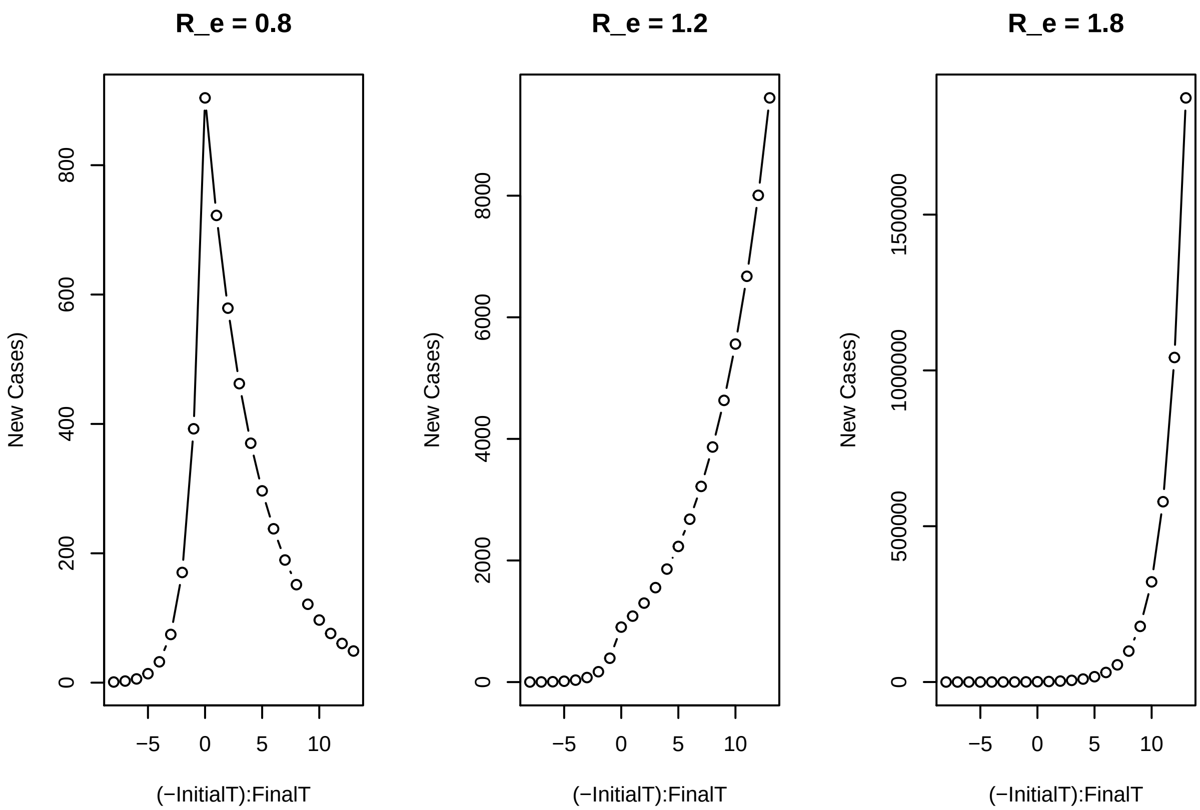
```
## [1] "The mean number of new cases today is 392.466"
```

```r
print(paste("The variance of new cases today is",
            var(Ncases[InitialT, 1, ])))
```

```
## [1] "The variance of new cases today is 361283.319482966"
```

2. For each strategy, plot the mean number of new cases after 1-52 weeks.

```r
par(mfrow = c(1,length(Re)))
for (i in 1:length(Re)) {
  plot((-InitialT):FinalT, rowMeans(Ncases[, i, ]), type = "b",
       ylab = "New Cases)", main = paste0("R_e = ", Re[i]))
}
```



3. For each strategy, what is the probability of zero new cases after 52 weeks?

Since the output of `rnbinom()` can be `NA` if `NPath0[t]` is too large, we only simulate up to 13 weeks after time 0 here.

```r
Ncases2 <- Ncases[ , , Ncases[8, 1, ] != 0]
for (i in 1:length(Re)) {
  print(paste("The probability of zero new cases under strategy", i, "is:",
              mean(Ncases2[(InitialT+FinalT+1), i, ] == 0)*100, "%."))
}
```

```
## [1] "The probability of zero new cases under strategy 1 is: 5.38720538720539 %."
## [1] "The probability of zero new cases under strategy 2 is: 0.673400673400673 %."
## [1] "The probability of zero new cases under strategy 3 is: 0.336700336700337 %."
```